附录：

Differential Geometry

1. 课堂讲授学时 Lecture Hours: 32
2. 课堂实验学时 Laboratory Hours: 0
3. 课下研讨学时 Colloquial Hours: 16
4. 学生课下投入学时 Individual Study Hours: 0
5. 学分 Credits: 2
6. 开课学年学期（如果有强制性的要求则 必须填， 否则可以不填） Occurrence: Summer Course
7. 先修课程 Prerequisite(s): Mathematical Analysis I, II, III, Advanced Algebra I, II, Ordinary Differential Equations* and Topology(*Recommended, not required as prerequisite)
8. 课程概要 Course Description: 100 字以内 ， 学习内容以学术 关键词出现。
   This course is built as a unified progression from the classical differential geometry of surfaces to modern global Riemannian geometry. In Part I, the domestic instructor develops four tightly integrated modules: extrinsic surface curvature (Gauss map, second fundamental form), intrinsic geometry (Theorema Egregium, geodesics), global surface theory (Gauss-Bonnet, minimal surfaces), and a transition to smooth manifolds. These foundations are then deepened in Part II by Prof. Tuschmann, who presents four advanced modules: comparison geometry (Rauch, Toponogov, Bishop-Gromov), nonnegative and positive curvature (Synge, Soul theorem, Gromov's Betti number estimate), collapsing geometry (Gromov-Hausdorff convergence, Cheeger-Fukaya-Gromov theory), and moduli spaces of Riemannian metrics (Tuschmann & Wraith 2015).
9. 课程预期学习成果 Course Outcomes:
   By the end of successful completion of this course, the student will be able to:
   (1). Parametrize regular surfaces, compute tangent planes, first and second fundamental forms, and principal curvatures; classify points via Gaussian and mean curvature.
   (2). Prove the Theorema Egregium and explain why Gaussian curvature is intrinsic.
   (3). Define geodesics on surfaces, derive the geodesic equations, and compute geodesics for standard metrics.
   (4). State and apply the local and global Gauss–Bonnet theorem, connecting curvature to Euler characteristic.
   (5). Translate surface theory into the language of smooth Riemannian manifolds (Riemannian metric, Levi-Civita connection, curvature tensor).
   (6). Apply the Rauch, Toponogov, and Bishop-Gromov comparison theorems to deduce consequences of curvature bounds.

(7). Describe Synge's theorem, the Soul theorem, and Gromov's Betti number estimate for manifolds of nonnegative/positive curvature.
(8). Outline Gromov-Hausdorff convergence, Cheeger-Fukaya-Gromov collapsing theory and its role in forming moduli spaces of metrics under curvature constraints.
(9). Explain and the construction of metric moduli spaces, with reference to Tuschmann & Wraith's book(2015).

10. 教学内容与学时分配 Course Content, Laboratories and Laboratory Hours（有则填，没有则不填），Colloquial Hours（有则填， 没有则不填）：

Part I: Surface and Manifolds

(1). Extrinsic Geometry of Surfaces (4 Class Hour)

- Course Content: Regular surfaces, tangent plane, First fundamental form, Gauss map, second fundamental form, normal curvature, principal curvatures, Gaussian and mean curvature
- Classroom 4 hours
- Colloquial Hours: 2

(2). Intrinsic Geometry of Surfaces (4 Class Hour)

- Course Content: Isometry, Theorema Egregium, parallel transport, geodesics, geodesic curvature, exponential map.
- Classroom 4 hours
- Colloquial Hours: 2

(3). Global Geometry of Surfaces (4 Class Hour)

- Course Content: Gauss-Bonnet theorem, rigidity of sphere, completeness, Bernstein theorem of minimal surfaces, CMC surfaces.
- Classroom 4 hours
- Colloquial Hours: 2

(4). From Surfaces to Manifolds (4 Class Hour)

- Course Content: Smooth manifolds, tangent bundle, Riemannian metrics, Levi-Civita connection, curvature tensor, sectional curvature, Ricci curvature, scalar curvature.
- Classroom 4 hours
- Colloquial Hours: 2

Part II: Global Riemannian Geometry

(1). Comparison Geometry (6 Class Hour)

- Course Content: Riemannian manifolds revisited; connections and geodesics; Jacobi fields and conjugate points; Rauch comparison theorem; Toponogov triangle comparison theorem; lower curvature bounds and global geometry; Bishop – Gromov volume inequality, Bonnet-Myers theorem.
- Classroom 4 hours
- Colloquial Hours: 2

(2). Nonnegative and Positive Curvature (4 Class Hour)

- Course Content: Manifolds with nonnegative curvature: Soul theorem (Cheeger – Gromoll), examples; Manifolds with positive curvature: Synge's theorem, Gromov's Betti number estimate, classification conjectures, open problems.
- Classroom 4 hours
- Colloquial Hours: 2

(3). Collapsing Geometry (4 Class Hour)

- Course Content: Gromov-Hausdorff convergence, collapsing with lower curvature bounds, fiberation theorem, collapsing and singular spaces.
- Classroom 4 hours
- Colloquial Hours: 2

(4). Moduli Spaces of Riemannian Metrics (2 Class Hour)

- Course Content: overview of Tuschmann & Wraith's book (2015).
- Classroom 4 hours
- Colloquial Hours: 2

考核与成绩评定 Grading:

Homework: 30%

Project: 30%

Group Presentation: 10%

Final Exam: 30%

11. 教材，参考书 Text & Reference Book:

(1). Jiagui Peng, Qing Chen, Differential Geometry, Second edition, Higher Education Press, 2017， ISBN: 978-7-04-056950-6.

(2). Manfredo P. do Carmo, Differential Geometry of Curves and Surfaces, Second edition, Dover Publications, 2016, ISBN: 978-0-486-80699-0.

(3). Wilderich Tuschmann, David J. Wraith, Moduli Spaces of Riemannian Metrics, Oberwolfach Seminars, Volume 46, 2015, ISBN 978-3-0348-0947-4.

12. 编写教师Course Lecturer: Chao Qian

编写教师 Course Lecturer（签字）： Chao Qian.